

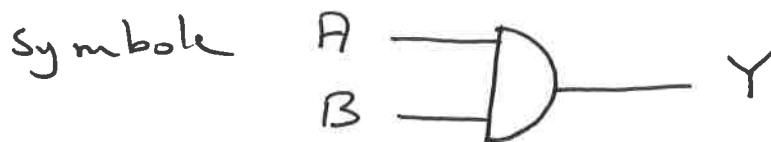
Binary Logic Gates:-

Electronic Logic gates are primary used in digital computer they are many factor mainly as IC the Trs, diode & any other solide state computer ts.

The gates its have one input or many input but its have one output. The (American standard Association) symbols are:-

- ① AND Gate.
- ② OR Gate.
- ③ Inverter Gate or (NOT Gate).
- ④ NAND Gate = Not AND
- ⑤ NOR Gate = Not OR

① AND Gate:-



- AND : Logical multiplication

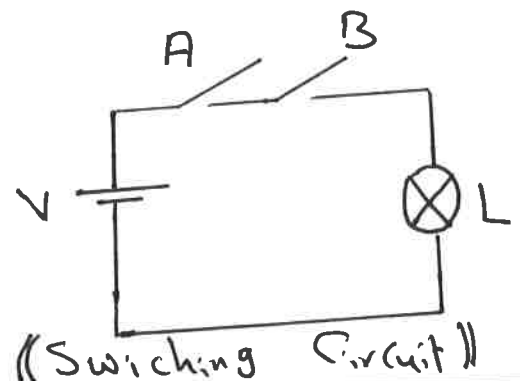
- No. of variable = 2

- No. of propanties = $2^2 = 4$

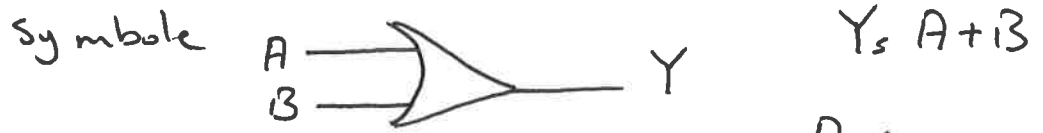
- Function : $F = Y = A \times B = A \cdot B = AB$

- Truth table

A ^{i/p}	B	Y ^{o/p}
0	0	0
0	1	0
1	0	0
1	1	1



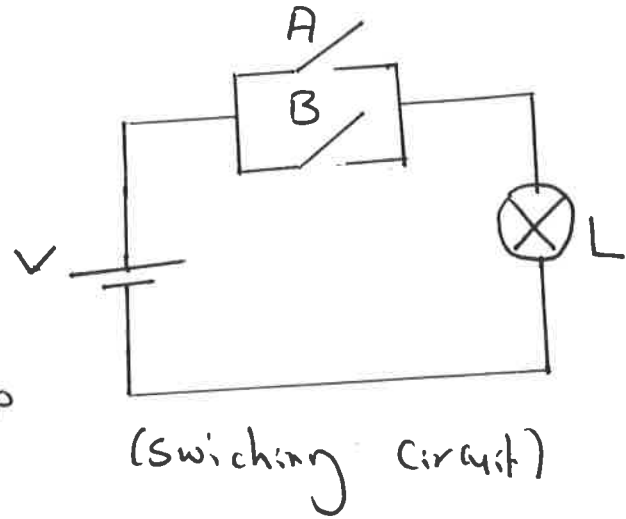
② OR Gate:-



- OR : Logical Addition.
- No. of variable ≤ 2
- No. of pro. $= 2^2 = 4$
- Function $Y = A + B$

Truth table

i/p	A	B	Y o/p
	0	0	0
	0	1	1
	1	0	1
	1	1	1



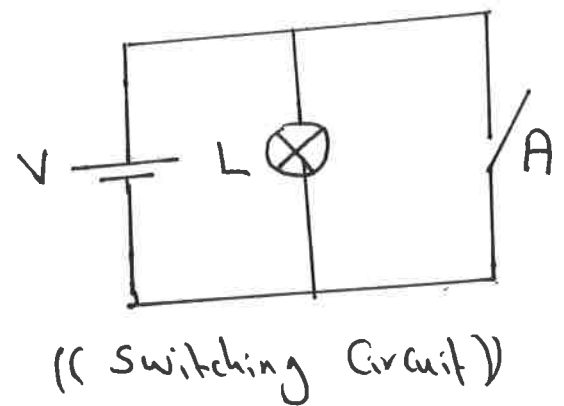
③ NOT Gate

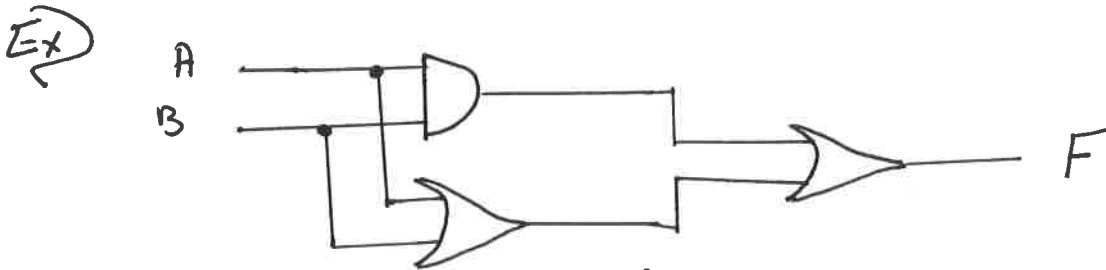


- Function $Y = \bar{A}$

Truth table

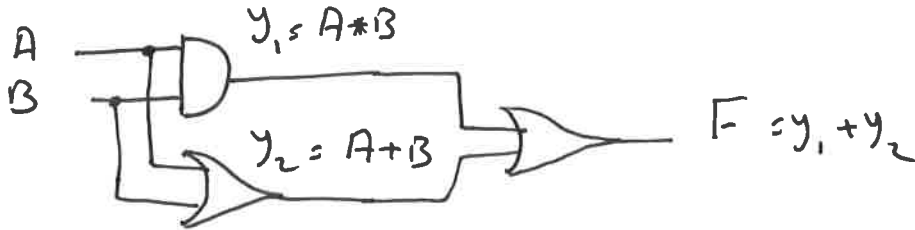
i/p	Y o/p
0	1
1	0





Find the truth table of F?

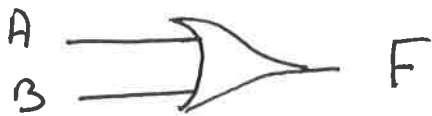
Solⁿ: —



$$\begin{aligned}
 F &= AB + A + B \\
 &= A(B+1) + B \\
 &= A * 1 + B \\
 &= A + B
 \end{aligned}$$

* Truth table

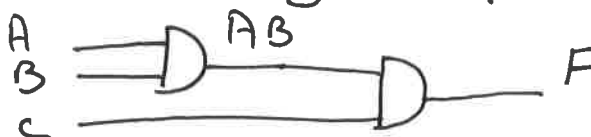
A	B	AB	A+B	F
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	1



Ex) Implement the following function :-



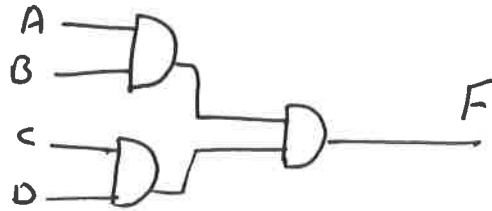
③ $F = ABC$ using 2-input AND gate only



④ $F = (AB)(CD)$



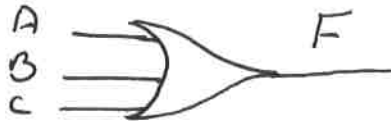
② using 2-input AND gate



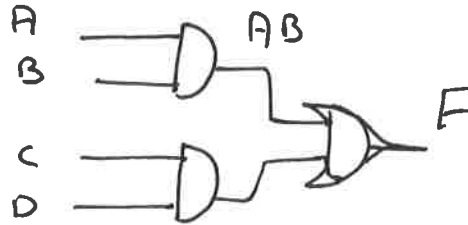
⑤ $F = A + B$



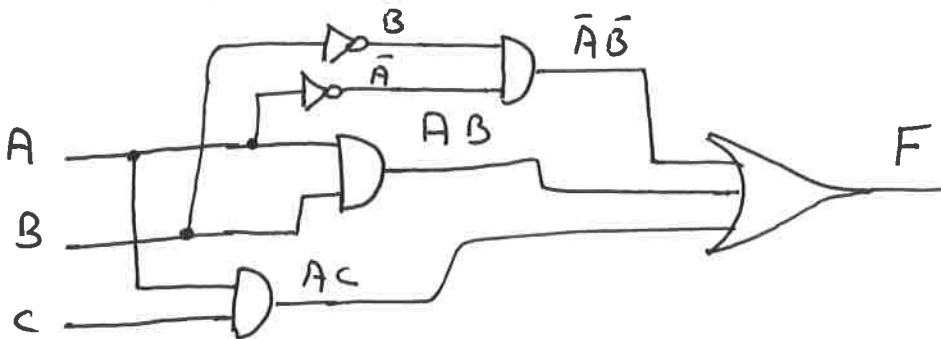
② $F = A + B + C$



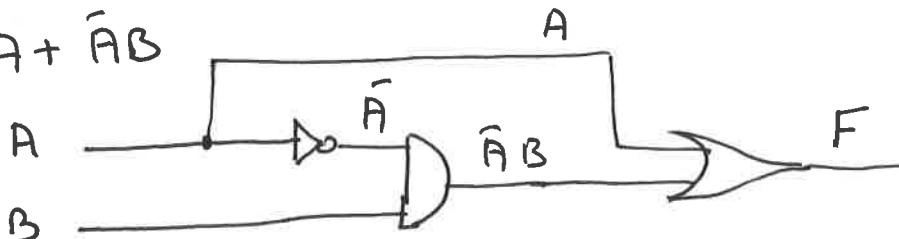
⑦ $F = (AB) + (CD)$



⑧ $F = AB + AC + \bar{A}\bar{B}$



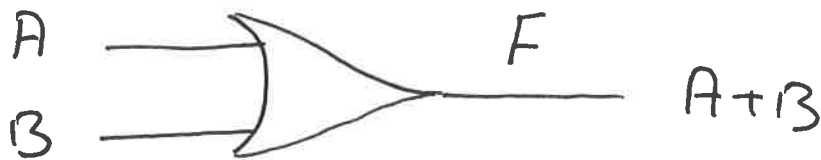
⑨ $F = A + \bar{A}B$





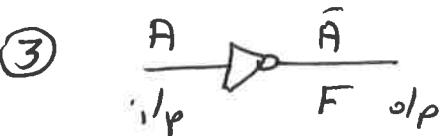
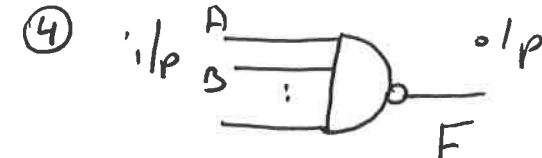
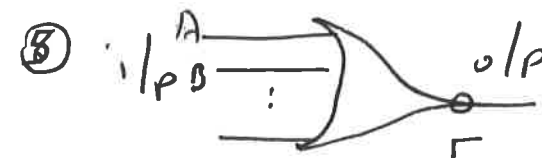
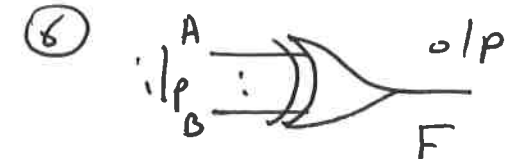
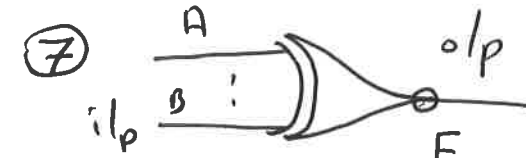
The truth table for the last Function

A	B	\bar{A}	$\bar{A}B$	F	(A+B)
0	0	1	0	0	0
0	1	1	1	1	1
1	0	0	0	1	1
1	1	0	0	1	1

$$\therefore A + \bar{A}B \equiv A + B$$



The American Standard Association (ASA) symbols for electronic logic gates:—

Symbol	Function	Comments
① 	$F = A \cdot B$	AND-gate
② 	$F = A + B$	OR-gate
③ 	$F = \bar{A}$	Not-gate
④ 	$F = \overline{A \cdot B}$	NAND-gate
⑤ 	$F = \overline{A + B}$	NOR-gate
⑥ 	$F = A\bar{B} + \bar{A}B$ $= A \oplus B$	EX-OR gate
⑦ 	$F = AB + \bar{A}\bar{B}$ $= A \odot B$	EX-NOR gate

Boolean Algebra:-

Def:- Any Collection $\{A: x+y; x \cdot y; \bar{x} \in A\}$
where $A: \text{Set}$

The operations $(+; \cdot; -)$ together with the following axioms is called Boolean Algebra.

Equivalent:-

2 Expressions are equivalent when one of them only equal to the other

$$\begin{aligned} \text{Ex)} \quad a \cdot b &= c \cdot 0 \\ 1 &\rightarrow 1 \\ 0 &\rightarrow 0 \end{aligned}$$

Complement:- 2 Expressions are Complement each other when one expression $= 1$ the other equal $= 0$, or when 1 expression $= 0$ the other $= 1$

The complement of A is \bar{A}

Rules of Complements:-

Two complement any (Boolean) expression change.

- ① all 0's to 1's.
- ② all 1's to 0's.
- ③ all (+)'s to (.)'s.
- ④ all (.)'s to (+)'s.

Then :-

Complement each variable

$$\text{Ex)} \quad F = A \cdot B + C$$

$$\bar{F} = \overline{A \cdot B + C}$$

$$\bar{F} = \bar{A} + \bar{B} \cdot \bar{C}$$

$$\text{Ex)} \quad F = \overline{A \bar{B} C D + A + B C}$$

$$\bar{F} = A \bar{B} C D + A + B C$$

$$\bar{F} = \bar{A} + B + \bar{C} + \bar{D} \cdot \bar{A} \cdot \bar{B} + \bar{C}$$

Duality :-

It states that every algebraic expression deducible from the postulates of Boolean Algebra remains valid if the operators and identity elements are interchanged.

Rules of Duality :-

change all 0's to 1's

change all 1's to 0's

change all (.)'s to (+)'s

change all (+)'s to (.)'s

but Don't Complement the Variable.

$$F = A \cdot B + \bar{A} \cdot B + C$$

$$\text{Dual} \quad F = A + B \cdot \bar{A} + B \cdot C$$

Ex) Find the Complement and the Dual of the following

- ① $A \cdot B$ ② $A + B$ ③ $A \cdot B \cdot C \cdot D$ ④ $A + B + C + D$

Solⁿ:-

	Complement	Dual
①	$\bar{A} + \bar{B}$	$A + B$
②	$\bar{A} \cdot \bar{B}$	$A \cdot B$
③	$\bar{A} + \bar{B} + \bar{C} + \bar{D}$	$A \cdot B \cdot C \cdot D$
④	$\bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D}$	$A + B + C + D$

Axioms of Boolean Algebra:-

① IF $x, y \in A$; Then $x + y \in A$; $x \cdot y \in A$; $\bar{x}, \bar{y} \in A$

② IF $x, y \in A$; Then $x \cdot y = y \cdot x$

③ $x, y, z \in A$ Then $x(y + z) = xy + xz$

④ IF $x \in A$ \exists $0 \in A$ \exists $1 \in A$ such that
 $x + 0 = x$
 $x \cdot 1 = x$

⑤ IF $x, y, z \in A$ Then $(x + y)z = (x + yz)$

⑥ IF $x \in A$ \exists $\bar{x} \in A$ such that
 $x \cdot \bar{x} = 0$
 $x + \bar{x} = 1$

Postulate:-

- ① $X + 0 = X$
- ② $X + \bar{X} = 1$ " Complementary Law "
- ③ $X(y+z) = Xy + Xz$
- ④ $X + y = y + X$
- ⑤ $X \cdot 1 = X$
- ⑥ $X \cdot \bar{X} = 0$
- ⑦ $X + yz = (X + y)(X + z)$
- ⑧ $y \cdot X = X \cdot y$

Theorems:-

- ① $X + X = X$
- ② $X + 1 = 1$
- ③ $X + Xy = X$
- ④ $X + \bar{X}y = y$
- ⑤ $z\bar{X}y = zX + zy$
- ⑥ $Xy + \bar{X}z + yz = Xy + \bar{X}z$
- ⑦ $X \cdot X = X$
- ⑧ $X \cdot 0 = 0$
- ⑨ $X(X + y) = X$
- ⑩ $\bar{\bar{X}} = X$

Ex) Simplify the following Expressions using Boolean Algebra expr. :-

$$\textcircled{1} AB + A(B+C) + B(B+C)$$

$$\textcircled{2} A + A \cdot \bar{B} + \bar{A} \cdot \bar{B}$$

$$\textcircled{3} A \cdot B + \bar{A}\bar{B} + A$$

Solⁿ:-

$$\begin{aligned} \textcircled{1} AB + A(B+C) + B(B+C) &\leq AB + AB + AC + B + BC \\ &\leq A(B+C) + B(1+C) \\ &\leq AB + AC + B \\ &\leq B(A+1) + AC \\ &\leq B + AC \end{aligned}$$

$$\begin{aligned} \textcircled{2} A + A \cdot \bar{B} + \bar{B} \cdot \bar{A} &\leq A(1 + \bar{B}) + \bar{A}\bar{B} \\ &\leq A + \bar{A}\bar{B} \\ &\leq B \end{aligned}$$

$$\textcircled{3} AB + \bar{A}\bar{B} + A = 1 + A$$

≤ 1

Ex) Simplify the following Function

$$F = A\bar{C} + ABC + A\bar{C}\bar{D} + CD$$

$$\leq A(\bar{C} + BC) + C(A\bar{D} + D)$$

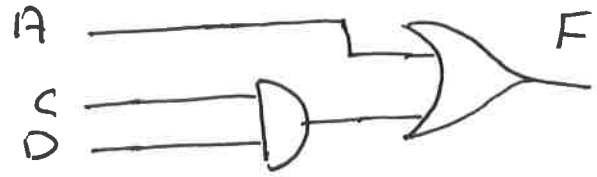
$$\leq A(\bar{C} + B) + C(A + D)$$

$$\leq A\bar{C} + AB + AC + CD$$

$$s \quad A(C + \bar{C}) + AB + CD$$

$$s \quad A + AB + CD$$

$$s \quad A + CD$$



Ex) Simplify the following Function:-

$$\textcircled{1} \quad F = AB + \bar{A}B\bar{C} + \bar{A}BC$$

$$\textcircled{2} \quad F = \bar{A}\bar{C} * (\bar{B} + BD) + A\bar{C}D$$

/ H.W

Ex) Simplify the following Function and Find the truth table?

$$F = A(B+C) + \bar{A}B + A\bar{C}$$

$$= AB + AC + \bar{A}B + A\bar{C}$$

$$= A(C + \bar{C}) + B(A + \bar{A})$$

$$F = A + B$$

A	B	C	\bar{A}	\bar{C}	B+C	A(B+C)	$\bar{A}B$	$A\bar{C}$	F	A+B
0	0	0	1	1	0	0	0	0	0	0
0	0	1	1	0	1	0	0	0	0	0
0	1	0	1	1	1	0	1	0	1	1
0	1	1	1	0	1	0	1	0	1	1
1	0	0	0	1	0	0	0	1	1	1
1	0	1	0	0	1	1	0	0	1	1
1	1	0	0	1	1	1	0	1	1	1
1	1	1	0	0	1	1	0	0	1	1